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Modeling Shocks in Periodic Lattice Materials

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Abstract. Periodic lattice materials are extremely light relative to their stiffness and strength. Developments in additive manufacturing technologies opens the possibility of using periodic lattices as energy absorbers for impact loading. This work extends an equivalent continuum material model for periodic, stretch dominated lattices to shock compression by augmenting the model with an equation for the evolution of relative density under volumetric plastic deformation. When compared to detailed finite element simulations, this simple modification to the equivalent continuum model accurately captures some parts of the shock response, especially the behavior of elastic precursors. However, the model is less accurate for the properties of the compaction shock, reflecting inaccuracies in the final state of the material.

INTRODUCTION

Additively manufactured lattice meta-materials show promise for future use in very light yet very stiff and strong structures [10]. Researchers have begun to develop models for the response of these materials, but most current models are geared towards quasi-static deformations [6, 3]. Recent work by the authors and others have extended such models along the lines described in [9] to accurately describe elastic wave propagation by incorporating microinertial effects. Like foams and similar underdense materials [1, 4], these lattice structures have the potential to be excellent impact and shock mitigation materials, as they have the ability to absorb large amounts of energy per unit weight. This work extends, in an approximate manner, an equivalent continuum model representing the behavior of stretch-dominated lattices for quasi-static and small-deformation dynamic behavior to account for shock compression.

BASE MATERIAL MODEL

The base material model describes the quasi-static and elastic/weakly plastic dynamic properties of lattice meta-materials as an equivalent continuum. The model is meant to treat a large region of material comprised of a periodic lattice. Further details are found in [7], this section briefly outlines the base model before extending it to describe the response of the lattice to weak shocks.

Limiting the model to small deformations and simple stretch-dominated lattices, the elasticity tensor of the equivalent medium is:

$$\mathbf{C} = E\bar{\rho}\frac{\sum_{s=1}^{n_{struts}} A_i l_i \mathbf{n}_i \otimes \mathbf{n}_i \otimes \mathbf{n}_i \otimes \mathbf{n}_i}{\sum_{i=1}^{n_{struts}} l_i A_i} \tag{1}$$

with E the Young's modulus, A_i the area, $\mathbf{n}_i$ the normal unit vector, and l_i the length of each strut in the unit cell and $\bar{\rho}$ the relative density of the lattice.

Plastic deformation can be described as an additive decomposition of plastic extension/compression in all the struts in a unit cell

$$\dot{\boldsymbol{\sigma}} = \bar{\rho}E\hat{\mathbf{C}} : \left(\dot{\boldsymbol{\varepsilon}} - \sum_{i=1}^{n_{struts}} \dot{\varepsilon}_i \left(\mathbf{n}_i \otimes \mathbf{n}_i\right)\right) \tag{2}$$

with $\dot{\boldsymbol{\sigma}}$ the stress rate tensor, $\dot{\boldsymbol{\varepsilon}}$ the total strain rate tensor, and $\dot{\varepsilon}_i$ the strain in each strut. We assume a power law relation between the stress in each strut and the corresponding plastic strain rate

$$\dot{\varepsilon}_i = \dot{\varepsilon}_0 \left| \frac{\sigma_i}{\bar{\sigma}_i} \right|^{n-1} \frac{\sigma_i}{\bar{\sigma}_i} \tag{3}$$

for σ_i the axial stress in the strut, $\dot{\varepsilon}_0$ a reference strain rate and $\bar{\sigma}_i$ the flow stress in the strut. The resolved axial stress in each strut is given by

$$\sigma_i = E\left(\mathbf{n}_i \otimes \mathbf{n}_i\right) : \mathbf{C}^{-1} : \boldsymbol{\sigma}. \tag{4}$$

Any 1D hardening model can describe the flow stress for an individual strut. Here we use simple linear hardening. Integrating these equations describes the quasi-static deformation of a lattice.

Capturing the dynamics of the material also requires corrections to the inertial body force of the equivalent medium. These corrections represent microinertial effects, reflecting the non-uniform distribution of inertia in the periodic lattice. Summarizing the derivation found in [7], the inertial body force in the equivalent medium is *not*

$$\mathbf{f}_{inertia} = \rho_B \bar{\rho} \mathbf{a} \tag{5}$$

with $\mathbf{a}$ the acceleration field and ρ_B the bulk density of the strut material, but rather

$$\mathbf{f}_{inertia} = \left\{ \rho_B \bar{\rho} \frac{\sum_{i=1}^{n_{struts}} l_i A_i \left(\mathbf{n}_i \otimes \mathbf{n}_i\right)}{\sum_{i=1}^{n_{struts}} l_i A_i} \right\} \cdot \mathbf{a} \tag{6}$$

That is, density is a matrix with some potential anisotropy, not an isotropic scalar. Without these corrections the dynamics of the equivalent continuum diverge substantially from the actual behavior of the periodic lattice.

EXTENSION TO WEAK SHOCKS

In the previous section the relative density parameter $\bar{\rho}$ remains fixed. Volumetric strain can change the density of the equivalent medium, but this change in density will not effect the form of the constitutive equations. This is a reasonable assumption for small deformations but unreasonable for impact loading where the material substantially densifies at the compaction shock front.

Consider the Eq. 2 – the stress/strain relation for the small-deformation model. At a given state of strain and material history the stress in the material scales linearly with relative density $\bar{\rho}$. A linear scaling between stiffness and relative density is also the experimentally observed relation for stretch-dominated lattices. Therefore, as a first attempt at extending the material model to deformation regimes suitable for studying the behavior of shocks, we allow the relative density $\bar{\rho}$ to evolve to represent the compaction associated with a shock front.

The relative density is $\bar{\rho} = V_m/V$ where V_m is the volume of the bulk material and V is the total volume. Assuming:

1. the change in density occurs entirely through elimination of void space, rather than compression of the bulk material
2. an additive decomposition of strain

yields the evolution equation

$$\dot{\bar{\rho}} = -\bar{\rho}\,\mathrm{tr}\,\boldsymbol{\varepsilon}_p \tag{7}$$

describing the evolution of relative density with plastic strain. This evolution equation supplements the small strain material model described above. The implementation of the model is a coupled integration of Eqs. 2 and 7.

This form of the model resembles a weak-shock version of the $p - \alpha$ formulation developed in [5] for granular ductile materials. By weak shocks we mean this model only considers the mechanical aspects of deformation. This implies the theory will be inaccurate for stronger shocks where thermodynamical effects cannot be neglected.

HUGONOIT CURVES

For the weak shock theory Hugoniot curves are essentially dynamic compaction stress-strain curves. In particular, for planar impact, given the relation $\sigma_{11}(\varepsilon_{11})$ a Hugoniot curve of any type can be derived using the Rankine conditions representing conservation of mass and momentum at the shock front

$$\begin{aligned} -(\varepsilon_{11}^{(2)} - \varepsilon_{11}^{(1)})U_s &= \dot{x}^{(2)} - \dot{x}^{(1)} \\ \rho_R U_s(\dot{x}^{(2)} - \dot{x}^{(1)}) &= -(\sigma_{11}^{(2)} - \sigma_{11}^{(1)}) \end{aligned} \tag{8}$$

where (1) is the material state before the shock and (2) is the material state after the shock, $\dot{x}$ is the material velocity, U_s the shock velocity, and ρ_R the reference density, here equal to $\bar{\rho}\rho_{bulk}$ with ρ_{bulk} the bulk density of the strut material [2]. The equivalent continuum Hugoniot curves in Fig. 2 are generated with this approach for the octet truss unit cell shown in Fig. 1. Material properties for the model are listed in Table 1, with the exception of the Poisson's ratio which does not come into the model formulation. These properties are typical of polymer material constructed via stereolithography.

For verification, Fig. 2 compares the model Hugoniot curve to Hugoniot points generated from finite element simulations. Figure 1 describes the model used in ALE3D – an arbitrary Lagrangian-Eulerian finite element package developed by Lawrence Livermore National Laboratory for hydrodynamic simulation [8]. The model represents the collision of a massive impactor on a periodic lattice. Material properties are the same as for the equivalent continuum model, shown in Table 1. As the figure shows, two compression fronts develop in the simulation. The first is an elastic precursor and the second is the compaction shock front associated with the densification of the lattice to its final state. Therefore, each impact velocity v_0 generates two points on the Hugoniot – a point associated with the Hugoniot elastic limit (HEL) and a point associated with the compaction shock. By measuring the material velocity before and after the fronts and the front speeds the jump conditions (Eq. 8) can generate the remainder of the data of interest.

As the figure shows, the augmented equivalent continuum model captures the elastic behavior of the lattice – the speed of the elastic precursor and the approximate position of the HEL. It is less accurate for the compaction shock, which involves regions of the material undergoing large deformations This is because of the assumption made to extend the model to large compaction – allowing the relative density to evolve with plastic strain but keeping the form of the model the same. Consider the limit of shock which fully compresses the void space out of the lattice. Physically at this point the Hugoniot should begin to follow the equation of state of the bulk material. However, this model would instead continue to deform as a lattice, albeit a lattice with $\bar{\rho} = 1$. This means, for example, that the elastic behavior of the fully dense material would continue to be anisotropic and the bulk modulus of the fully dense lattice would not match the bulk modulus of the strut material. Additionally, as mentioned previously, the general mechanical assumptions of the work presented here limits the applicability of the model to weak shocks where thermodynamic effects may be neglected.

With these caveats, the model still captures the speed of a shock front traveling through the lattice with reasonable accuracy. Particular at larger impact velocities, the slopes of Rayleigh lines connecting the HEL and the impact velocity do not vary significantly between the model and the finite element simulations. It less accurately captures the amount of energy dissipated by the shock – the area between the Hugoniot and a Rayleigh line.

Using the continuum model to calculate properties for impact on the lattice is much less computationally expensive than calculating similar properties with the finite element simulation. For comparison, the finite element simulations have more than 10 million elements, while determining the Hugoniot from the continuum model is essentially a material point calculation. Figure 2 compares the model response for impact in three different directions. Both the elastic properties (sound speed and HEL) and shock properties (shock speed and dissipated energy) are anisotropic. In particular the 110 and 111 directions have a faster longitudinal elastic sound speed, a lower shock speed, and a higher dissipated energy than for impact in the 100 direction.

CONCLUSIONS

Incorporating an evolution equation for the relative density extends the equivalent continuum model for stretch dominated, periodic lattices to represent the shock compaction response of the material. However, this simple modification to the equations fails to capture behavior near the shock front, where the material collapses and looses its lattice structure. Near the shock, the lattice more resembles solid material interspersed with voids. Future work will consider representing this portion of the response with a porous plasticity model and thereby better capture the shock compression response of periodic lattice materials.

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TABLE 1. Material properties used for both the finite element and the equivalent continuum simulations.

Property	Description	Value
E	Young's modulus	1780 MPa
ν	Poisson's ratio	0.35
σ_0	Yield stress	40 MPa
H	Hardening modulus	100 MPa
ρ_B	Bulk density	1.18 g/cm^3

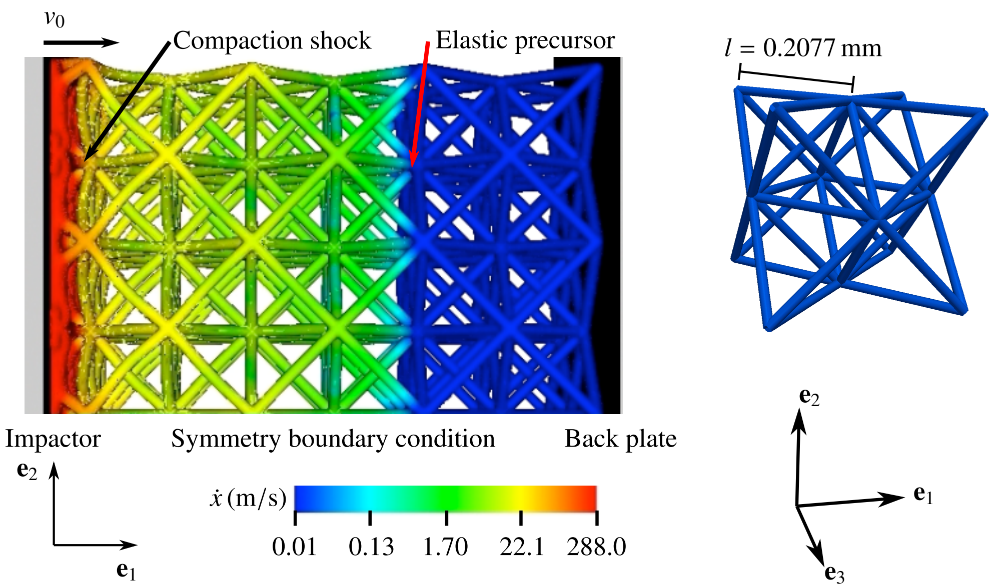

FIGURE 1. Figure showing a finite element simulation used to verify the equivalent continuum model for shock compression. The fringe colors show the material velocity in the *x*-direction. The logarithmic scale shows the elastic precursor and the compaction shock on the same figure. The compaction shock has some finite rise time – associated spatially with approximately the width of a single unit cell. The unit cell dimensions and directions are labeled on the right.

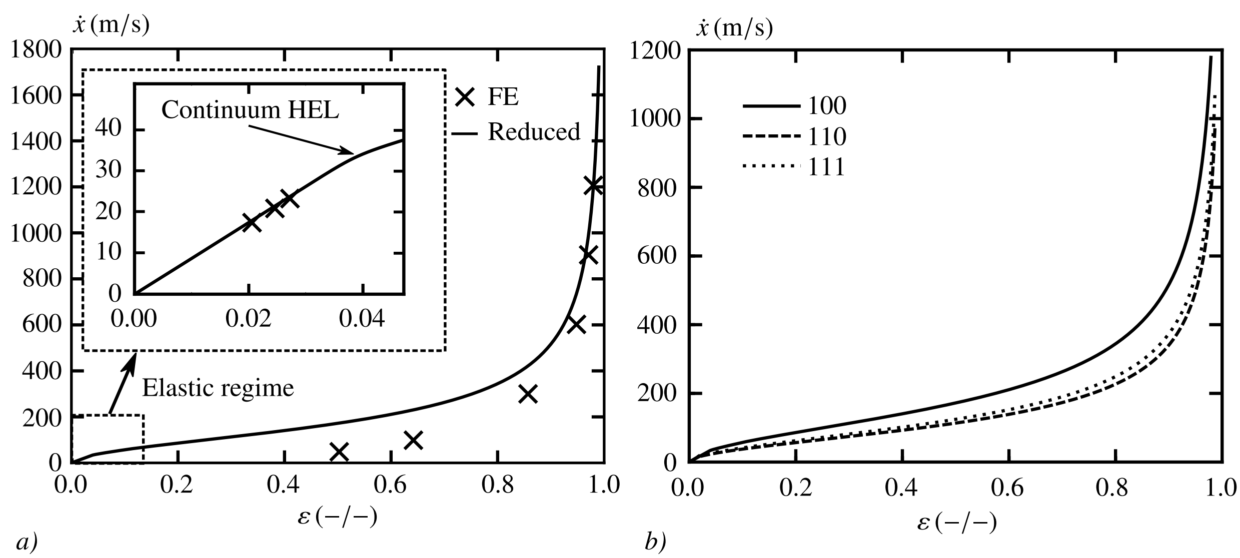

FIGURE 2. Material velocity-strain Hugoniots for the octet lattice. *a)* Plot comparing the continuum model results to the finite element simulations. *b)* Continuum model results for impact on the periodic octet in three directions. The 100 direction indicates impact in the $\mathbf{e}_1$ direction, 110 in the $\mathbf{e}_1 + \mathbf{e}_2$ direction, and so on (see Fig. 1).